**Make Sense of Problems Persevere in Solving Them**

  *Mathematically proficient students:*

|  |  |  |
| --- | --- | --- |
| **Begin by…*** **…explaining the problem to themselves, looking for entry points.**
* **…analyzing what is given/not given, analyzing relationships, determining goals of the problem.**
* **…making conjectures about solutions, planning solution pathways.**
* **…considering similar/simpler problems, trying special cases.**
 | **Solve by…*** **…monitoring and evaluating progress, changing course if necessary.**
* **…creating, then comparing and explaining connections between equations, tables, graphs.**
* **…drawing diagrams of important relationships.**
* **…searching for regularity or trends.**
 | **Finalize by…*** **…checking answers to problems using different methods.**
* **…continually asking, “Does this make sense?”**
* **…understanding approaches of others to solving complex problems**
* **…identifying connections and comparing different approaches.**
 |

**Reason Abstractly and Quantitatively**

 *Mathematically proficient students:*

|  |  |
| --- | --- |
| * **MAKE SENSE of Quantities (amounts, numbers, sizes) and their relationships in problem situations by…**
 | * **REASON Quantitatively (with amounts, numbers, sizes) to create a clear representation of the problem by…**
 |
| **Contextualizing:*** **…giving the numbers a context (background, labels, situations, locations).**
* **…pausing during numeric or symbolic manipulation to probe into the background or situation.**

**De-contextualizing:** * **…representing a problem using abstract numbers and symbols.**
* **…manipulating symbols outside a problem context.**
 | * **…considering the units (labels) for the numbers (i.e., yards or miles).**
* **…attending to the meaning of quantities, not just manipulating the numbers or symbols.**
* **…flexibly using different properties of operations and objects.**
 |

**Construct Viable Arguments Critique Others’ Reasoning**

 *Mathematically proficient students:*



|  |  |
| --- | --- |
| **Construct:*** **Construct arguments using previously established assumptions, definitions and results.**
* **Make conjectures. Build a logical progression of statements to explore the truth of conjectures.**
* **Analyze situations by breaking them into cases. Recognize and use counterexamples.**
* **Justify conclusions. Communicate conclusions to others.**
* **Reason inductively (generalize from individual cases). Make plausible arguments involving contexts.**
 | **Critique:*** **Listen to or read arguments of others, decide whether they make sense, ask questions to clarify or improve the arguments.**
* **Compare the effectiveness of two arguments.**
* **Distinguish correct logic or reasoning from that which is flawed.**
* **Explain a flaw in an argument.**
 |

**Model with Mathematics**

 *Mathematically proficient students:*



|  |  |
| --- | --- |
| **Use models to *solve problems* in everyday life, society, and work.****For Example:*** **In sixth grade, model a proportion or percentage problem by drawing bars.**
* **In middle grades, apply proportional reasoning models to plan a school event or analyze a community problem.**
* **In high school, use geometric models to solve a design problem or use a function to show a real-world relationship.**
* **In real-life, model a scheduling problem using a network.**
* **Identify, map and interpret relationships in real-world situations using diagrams, two-way tables, graphs, flowcharts, formulas, etc.**
 | **Use models to *understand and communicate* mathematical ideas.*** **Create a model to show the meaning of a percentage problem or an equation.**
* **Move interchangeably among models: that is concrete manipulatives, drawings, equations, tables, graphs, flowcharts, formulas.**
* **Abstract the mathematics from the models or create the models to explain the mathematics.**
* **Use models to simplify a situation and to make assumptions and approximations.**
* **Analyze and draw conclusions about mathematical relationships using diagrams, tables, graphs, flowcharts.**
* **Reflect on results and improve the model if needed.**
 |

**Use Appropriate Tools Strategically**

 *Mathematically proficient students:*



|  |  |  |
| --- | --- | --- |
| **At all age levels:*** **Consider available tools when solving a mathematical problem (pencil, paper, concrete model, ruler, protractor, calculator, spreadsheet, computer software).**
* **Make sound decisions about when tools might be helpful.**
* **Recognize insight to be gained and limitations of tools. For example, students detect possible errors (from graphing calculators) using estimation and other mathematical knowledge.**
 | **C:\Documents and Settings\CurricStaff\Local Settings\Temporary Internet Files\Content.IE5\C5QF0TEJ\MC900229685[1].wmf** | **At appropriate age levels:*** **Use technology to visualize results of assumptions, explore consequences, and compare predictions.**
* **Identify relevant mathematical resources, such as digital content on a website, and use them to pose or solve problems.**
* **Use technological tools to explore and deepen their understanding of concepts.**
 |

**Attend to Precision**

 *Mathematically proficient students:*



|  |  |
| --- | --- |
| **Communicate by…*** **…speaking, reading, and writing precisely to others.**
* **…using clear definitions in discussion with others and in their own reasoning.**
* **…stating the meaning of the symbols used, including using the equal sign consistently and appropriately.**
* **…specifying units of measure carefully.**
* **…labeling axes with units of measurement to clarify relationships of quantities in a problem.**
 | **Calculate and Problem Solve by…*** **…calculating accurately and efficiently.**
* **…expressing numerical answers with precision appropriate for the problem context.**
* **…making certain solution matches the problem asked.**
* **…estimating to check if computed solution is reasonable and therefore precise.**
 |

**Look for and Make**

**Use of Structure**

 *Mathematically proficient students:*



|  |  |
| --- | --- |
| * **Look closely to observe patterns or structure, for example:**
* ***Young students* might see that three + seven more is the same amount as seven + three more, or sort shapes according to number of sides.**
* ***Later*, students will see 7 x 8 equals (7 x 5) + (7 x 3), in preparation for the Distributive Property.**

* **In the expression *x*2 + 9*x* + 14, *older students* can see the 14 as (2 x 7) and the 9 as (2 + 7).**
* **Step back for a big picture look and shift perspective.**
* **See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.**
 |  |

**Look for, Express Regularity**

**in Repeated Reasoning**

 *Mathematically proficient students:*



|  |  |
| --- | --- |
| * **Notice if calculations are repeated and look both for general methods and for shortcuts, for example:**
* ***Upper elementary students* might notice when dividing 25 by 11 that they are repeating the same calculations over and over and recognize a repeating decimal.**
* ***Middle school students* might use a graphing calculator to observe a series of lines in slope/intercept form (*y*=m*x*+b), paying attention to what happens when “m” or “b” change, then drawing conclusions.**
* ***High school students* might notice the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead students to the general formula for the sum of a geometric series.**
 | * **Work to solve a problem—keep the problem in mind while working on details.**
* **Continually evaluate reasonableness of intermediate results during the problem solving process.**

 |