**Chapter 8 notes**

**The Binomial Distribution**

There are a few very common types of discrete probability distributions. One of the most important is called the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Properties of a binomial experiment:

 1. There are a fixed number of observations, called trials (n = # of trials).

 2. There are only 2 outcomes for each trial, success (S) or failure (F).

 3. Outcomes of different trials are independent.

 4. The probability of success, *p*, is the same for all trials

 Note: Other books use “π” instead of “p” for this probability.

The binomial random variable x is defined as:

 x = the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ observed when the experiment is performed

The probability distribution of x is called the **binomial** probability distribution.

 Notation: x ~ B(n, *p*)

\**The most important skill for using binomial distributions is the ability to recognize situations to which they apply and don’t apply.*

Are the following binomial? If not, why not?

1. x = # of heads in 3 tosses
2. x = # of rolls until you get a 6
3. x = # of free throws made in 5 tries
4. x = # of questions correct when randomly guessing on a test with 10 T/F and 10 MC questions (5 choices)

Suppose that 70% of people in the mall will make a purchase and 3 mall visitors are randomly selected. Use a tree diagram to identify the 8 possible outcomes and their probabilities.

Let x = the number of people who make purchases among the 3. Find the probability distribution of x.

What is the probability that 2 of the 3 mall visitors made a purchase?

Is there an easier way?

 

The binomial coefficient is the number of ways of arranging k successes among n observations is given by:

  = “n choose k” = 

*(TI84: Math: Prb) = nth row in Pascal’s triangle*

If x ~ B(n, *p*) and k = # of successes:

  for k = 0, 1, 2, ....., n

Suppose you rolled a die 4 times and let x = number of sixes. Find the probability distribution of x and the probability that you get 2 or more sixes.

**Using the TI-84 for the Binomial**

Suppose that you are a telemarketer and historically only 15% of people you call will remain on the phone for at least 1 minute. What is the probability that 1 of the next 5 people you call will remain on the phone for more than 1 minute?

Using the TI-84 to do binomial calculations:

Shortcuts on TI84: Distribution Menu (DISTR = 2nd VARS)

Assume x ~ B(5, .15)

Dist: binompdf(n, *p*, k) gives the probability that there will be k successes in n trials.

 ex: P(x = 1) = binompdf(5, .15, 1) =

Note: pdf stands for probability density function

Dist: binompdf(n, *p*) gives the probability of each possible value of k (0, 1, …, n)

 ex: binompdf(5, .15) =

Dist: binomcdf(n, *p*, k) gives the probability that the number of successes is  k

 ex: P(x ≤ 1) = binomcdf(5, .15, 1) =

Note: cdf stands for cumulative probability density function (cumulative means added up)

Dist: binomcdf(n, π) gives the cumulative probabilities for each possible value of k (0, 1, …, n)

 ex: binomcdf(5, .15) =

If I roll a die 30 times, what is the probability that I get at least 5 sixes?

What is the probability I get at least 2 sixes and at most 7 sixes?

**The mean and standard deviation of a binomial random variable:**

If x is a binomially distributed random variable with n trials and probability of success *p*, then

 µx = *np*andσx = *np(*1*-p)*

Suppose that 75% of all customers at a gas station choose 87 octane gas. If you observe the next 50 customers,

1. How many customers do you expect to choose 87 octane gas?
2. What is the standard deviation of the number of customers that choose 87 octane gas?
3. What is the probability that the number of customers who choose 87 octane gas is within 1 standard deviation of the mean?

**The normal approximation to the binomial distribution**

Suppose we were to roll a die one million times. What is the probability that we would get more than 167000 sixes?

In many cases, it is useful to approximate a discrete distribution with a continuous distribution. In particular, we often use the normal distribution to approximate the binomial distribution. Historically, this is because we didn’t have calculators with binomcdf. Even now, there are some problems even too formidable for our TI’s! Fortunately, the normal distribution approximates the binomial quite well in certain circumstances.

It seems like this approximation works best when *p* is*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.*

Rule: If x ~ B(n, *p*) and n *p* >10 and n(1- *p*)>10, then x is approximately N\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Note: n *p* = expected number of successes and n(1- *p*) = expected number of failures

 

Please note: We can only use this approximation if **n*p* >10.**

(Even though our calculator can handle any reasonable sample size, we need to use the normal approximation to the binomial when we do the inference procedures for proportions in later chapters.)

Suppose that 51% of US residents are women. In a random sample of size 1000, what is the probability that fewer than 500 women are selected?

In a sample of size 10, what is the probability that fewer than 5 women are selected?

**The Geometric Distribution**

In a binomial distribution, we are interested in the number of successes in a fixed number of trials. However, sometimes we are not interested in the number of successes, but rather, how long it will take to achieve one success.

 ex: x = number of children in a family that stops having children when they get their first boy

Variables of this sort are called \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Properties of a geometric experiment:

 1. Trials are performed until a success is achieved.

 2. There are only 2 outcomes for each trial, success (S) or failure (F).

 3. Outcomes of different trials are independent.

 4. The probability of success, *p*, is the same for all trials.

The \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ tells us the probability that it takes a specific number of Bernoulli trials to achieve a success. The geometric random variable is always:

 x = the number of trials required to get one success

The model is completely determined by one parameter (p) and is denoted: G(p).

For example, suppose I am rolling a die until I get a “5”. Is this Geometric?

What is the probability that I get a “5” on my first roll?

What is the probability that I get my first “5” on the 2nd roll

What is the probability that I get my first “5” on the 3rd roll

What is the probability that I get my first “5” on the kth roll?

For geometric models in general,

* P(X = x) = (1 – *p*)n-1 *p*

Suppose that 10% of the boxes of a certain cereal have a coupon for a free gallon of milk. If I were to buy only that brand of cereal,

1. What is the probability that I won’t get a coupon until the 20th box?
2. What is the probability that it will take at least 5 boxes to get the first coupon?

**Mean of a Geometric Distribution**

If the probability of getting a coupon is .1, then you would expect to get one coupon for every 10 boxes. Thus, you should expect that it takes 10 boxes to get 1 coupon.

If x ~ G(*p*), then:

* Expected Value: 
* Standard Deviation:  *(note: q = 1 – p)*

Note: This is called the Geometric Distribution since the probabilities form an infinite geometric series with a sum of 1.

Suppose that 30% of UHS students have a driver’s license. On average, how many students will you need to randomly select to find one with a DL? What is the probability it will take less students than expected?

Suppose that 10% of regular M&M’s are green and you randomly select M&M’s until you get a green M& M.